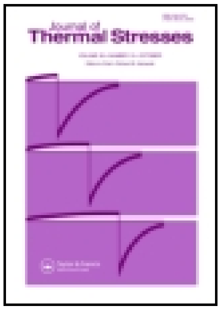

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LONGITUDINAL WAVE PROPAGATION IN A GENERALIZED THERMOELASTIC CYLINDER

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LONGITUDINAL WAVE PROPAGATION IN A GENERALIZED THERMOELASTIC CYLINDER

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In this paper the longitudinal wave propagation in a circular infinite cylinder is studied. The infinite circular cylinder is assumed to be made of a generalized thermoelastic material. The dispersion relation is obtained for the case in which the temperature is kept constant on the surface of the cylinder. Because of the complexity of the dispersion relation, the numerical solutions are given. For various values of parameters appearing in the field equations, some dispersion, attenuation, and phase velocity diagrams are presented.

INTRODUCTION

Longitudinal wave propagation in an infinite circular cylinder has been studied by Şuhubi [1], using the theory of coupled thermoelasticity. In that paper the dispersion relations were obtained for two particular cases and the resulting equations were solved analytically for small radii and weak coupling.

The present paper deals with the longitudinal wave propagation in an infinite circular cylinder, which is assumed to be made of a generalized thermoelastic material. The theory of generalized thermoelasticity, in which the entropy flux and the entropy source are determined by constitutive relations, allows us to include the temperature rates among the constitutive variables. It thus permits a finite velocity of propagation of thermoelastic disturbances. In this context the theory of generalized thermoelasticity was first proposed by Müller [2]. A simpler version was introduced by Green and Laws [3], Green and Lindsay [4], and independently by Şuhubi [5]. The last form of the theory will be employed in this paper.

We suppose that the lateral surface of the cylinder is free from stresses and is held at a constant ambient temperature. The frequency equation corresponding to this case is obtained quite easily. Because of the difficulty of solving this equation analytically, a numerical solution is given. Since there is an odd number of time derivatives in the field equations, the frequency equation $f(\Omega,k) = 0$, where Ω and k are nondimensional frequency and wave number, usually has complex roots, imag-

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inary parts of which measure the decay of waves due to thermoelastic interactions. Graphs of (Re Ω,k), (Im Ω,k), and (c/c_0,k) are plotted for various values of the material parameters. Here c is the phase velocity, and $c_0 = (E/\rho_0)^{1/2}$, where E is the Young modulus and ρ_0 is the density of the medium, is a reference velocity. Our results are similar to those given by Davies [6] for the purely elastic case.

FIELD EQUATIONS

In the absence of body forces the basic equations for temperature rate-dependent linear, isotropic thermoelastic medium can be written as follows [5]:

$$(\lambda + 2\mu)\, \nabla\nabla \cdot \mathbf{u} - \mu\, \nabla \times \nabla \times \mathbf{u} - \beta\, \nabla(T + \nu\dot{T}) = \rho_0 \ddot{\mathbf{u}} \tag{1a}$$

$$\rho_0 c_\nu \kappa\, \nabla^2 T = \rho_0 \tau \ddot{T} + \rho_0 c_\nu \dot{T} + \beta T_0\, \nabla \cdot \dot{\mathbf{u}} \tag{1b}$$

where $\mathbf{u}$ is the displacement vector, T is the temperature change about an equilibrium temperature T_0, λ and μ are Lamé's constants, ρ_0 is the density of the medium, κ is the diffusivity, c_ν is the specific heat per unit mass at constant strain ($K = \rho_0 c_\nu \kappa$ is the coefficient of thermal conductivity), β is a coupling factor that couples the heat conduction and elastic field equations, and ν and τ are constants introduced by the theory of generalized thermoelasticity.

If we now make use of the Helmholtz resolution for the displacement vector $\mathbf{u}$ as

$$\mathbf{u} = \nabla\varphi + \nabla \times \boldsymbol{\psi} \qquad \nabla \cdot \boldsymbol{\psi} = 0 \tag{2}$$

the field equations (1a) and (1b) become

$$(\lambda + 2\mu)\, \nabla^2\varphi - \beta(T + \nu\dot{T}) = \rho_0\ddot{\varphi} \tag{3a}$$

$$\mu\, \nabla^2\boldsymbol{\psi} = \rho_0\ddot{\boldsymbol{\psi}} \tag{3b}$$

$$\rho_0 c_\nu \kappa\, \nabla^2 T = \rho_0 \tau \ddot{T} + \rho_0 c_\nu \dot{T} + \beta T_0\, \nabla^2\varphi \tag{3c}$$

It is clearly seen that the rotational part of $\mathbf{u}$ is decoupled from the other parts. Using Eqs. (3a) and (3c), it is found that φ satisfies the following equation:

$$\left(\frac{\tau}{c_\nu}\frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} - \kappa\, \nabla^2\right)(\ddot{\varphi} - c_1^2\, \nabla^2\varphi) - \frac{\beta^2 T_0}{\rho_0^2 c_\nu}\, \nabla^2(\dot{\varphi} + \nu\ddot{\varphi}) = 0 \tag{4}$$

If a similar operation is carried out for the temperature T, it can be seen that the same equation is satisfied by T:

$$\left(\frac{\tau}{c_\nu}\frac{\partial^2}{\partial t} + \frac{\partial}{\partial t} - \kappa\, \nabla^2\right)(\ddot{T} - c_1^2\, \nabla^2 T) - \frac{\beta^2 T_0}{\rho_0^2 c_\nu}\, \nabla^2(\dot{T} + \nu\ddot{T}) = 0 \tag{5}$$

Here $c_1^2 = (\lambda + 2\mu)/\rho_0$ is the irrotational velocity.

Since we are dealing with the longitudinal oscillations of a circular cylinder, the displacement vector becomes $\mathbf{u} = (u_r, 0, u_z)$ in cylindrical polar coordinates (r,θ,z) and u_r, u_z, and T are functions of only r, z, and t. Taking this into account, Eqs. (4) and (5) take the following form for φ and T in cylindrical polar coordinates:

$$\left\{\left[\frac{\tau}{c_v}\frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} - \kappa\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)\right]\left[\frac{\partial^2}{\partial t^2} - c_1^2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)\right]\right.$$
$$\left. - \frac{\beta^2 T_0}{\rho_0^2 c_v}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial}{\partial t} + \nu\frac{\partial^2}{\partial t^2}\right)\right\}(T,\varphi) = 0 \tag{6}$$

LONGITUDINAL WAVES IN AN INFINITE CIRCULAR CYLINDER

Since we are investigating longitudinal wave propagation in a circular infinite cylinder, we assume that all field quantities are harmonic functions of z and t:

$$T = \Theta(r)\, e^{i(qz+\omega t)} \tag{7a}$$

$$\varphi = \Phi(r)\, e^{i(qz+\omega t)} \tag{7b}$$

$$\psi = \Psi(r)\, e^{i(qz+\omega t)} \tag{7c}$$

Putting Eqs. (7*a*)–(7*c*) into Eq. (6) and using the following dimensionless quantities:

$$\rho = \frac{r}{a} \qquad \zeta = \frac{z}{a}$$

where a is the radius of the cylinder, we obtain the following equation for Θ and Φ:

$$\left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} + \lambda_1^2\right)\left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} + \lambda_2^2\right)\{\Theta,\Phi\} = 0 \tag{8}$$

where λ_1^2 and λ_2^2 are the solutions of the following algebraic equation:

$$\lambda^4 - (\Omega^2 - i\alpha\Omega - i\varepsilon_1\Omega + \varepsilon_1\varepsilon_3\Omega^2 - 2k^2)\lambda^2 - k^2(\Omega^2 - i\alpha\Omega - i\varepsilon_1\Omega + \varepsilon_1\varepsilon_3\Omega^2)$$
$$- (i\alpha\Omega^3 - \varepsilon_2\Omega^4) + k^4 = 0 \tag{9}$$

The dimensionless quantities appearing in Eq. (9) are defined in the following manner:

$$qa = k \qquad \frac{a\omega}{c_1} = \Omega \qquad \frac{c_1 a}{\kappa} = \alpha$$

$$\varepsilon_1 = \frac{T_0 a}{\rho_0^2 c_v c_1 \kappa} \beta^2 \qquad \varepsilon_2 = \frac{c_1^2}{c_v \kappa} \tau \qquad \varepsilon_3 = \frac{c_1}{a} \nu \tag{10}$$

Since the parameter ε_1 defined in Eq. (10) couples the equations corresponding to the elastic wave propagation and the heat conduction, it can be called the *coupling factor*. The coefficient ε_2, which is introduced by the theory of generalized thermoelasticity, may render the governing system of equations hyperbolic. The parameter ε_3 is the coefficient of the term indicating the difference between empirical and thermodynamic temperatures.

The solution of Eq. (8) for Φ, which is finite for $\rho = 0$, is

$$\Phi(\rho) = AJ_0(\lambda_1\rho) + BJ_0(\lambda_2\rho)$$

where $J_0(\ )$ is a Bessel function of the first kind of order zero. We can thus write

$$\varphi = [AJ_0(\lambda_1\rho) + BJ_0(\lambda_2\rho)]\ e^{i(qz+\omega t)} \tag{11}$$

By changing arbitrary coefficients, the same solution can be written for T:

$$T = [CJ_0(\lambda_1\rho) + DJ_0(\lambda_2\rho)]\ e^{i(qz+\omega t)} \tag{12}$$

Of course, the coefficients in Eqs. (11) and (12) are not entirely independent of each other, because of Eqs. (3*a*) and (3*c*). Using Eq. (3*a*), the relations between those coefficients can be written in the following form:

$$C = \frac{\Omega^2 - k^2 - \lambda_1^2}{1 + i\Omega\varepsilon_3} \frac{\rho_0 c_1^2}{\beta a^2} A \tag{13a}$$

$$D = \frac{\Omega^2 - k^2 - \lambda_2^2}{1 + i\Omega\varepsilon_3} \frac{\rho_0 c_1^2}{\beta a^2} B \tag{13b}$$

Then φ and T become

$$\varphi = [AJ_0(\lambda_1\rho) + BJ_0(\lambda_2\rho)]\ e^{i(qz+\omega t)} \tag{14a}$$

$$T = \frac{\rho_0 c_1^2}{\beta a^2(1 + i\varepsilon_3\Omega)} [A(\Omega^2 - k^2 - \lambda_1^2)J_0(\lambda_1\rho) + B(\Omega^2 - k^2 - \lambda_2^2)J_0(\lambda_2\rho)]\ e^{i(qz+\omega t)} \tag{14b}$$

The solution for the vector potential $\boldsymbol{\psi}$ can be sought as follows (Eringen and Şuhubi [7]):

$$\boldsymbol{\psi} = \psi\mathbf{e}_z - l\frac{\partial\chi}{\partial r}\mathbf{e}_\theta$$

where ψ and χ are scalar functions and l is a scale parameter used to make both terms of the same dimension. Because of the nature of the problem, ψ is equal to zero. If l is chosen as a, the solution is

$$\psi = -a \frac{\partial \chi}{\partial r} \mathbf{e}_\theta$$

where χ satisfies the scalar wave equation

$$\nabla^2 \chi = \frac{1}{c_2^2} \frac{\partial^2 \chi}{\partial t^2} \tag{15}$$

where $c_2^2 = \mu/\rho_0$ is the velocity of shear waves. We look for a solution to Eq. (15) as follows:

$$\chi = \bar{\chi}(\rho)\, e^{i(qz+\omega t)}$$

Then the solution can be written

$$\chi = C J_0(\lambda_3 \rho)\, e^{i(qz+\omega t)}$$

where $\lambda_3^2 = \gamma^2 \Omega^2 - k^2$, $\gamma = c_1/c_2$, and C is an arbitrary constant. Thus the vector potential ψ is found as

$$\psi = C \lambda_3 J_1(\lambda_3 \rho)\, e^{i(qz+\omega t)} \mathbf{e}_\theta \tag{16}$$

After calculating the curl of ψ and the gradient of φ, we can write the components of $\mathbf{u}$ in the following form:

$$u_r = \left[-\frac{A}{a} \lambda_1 J_1(\lambda_1 \rho) - \frac{B}{a} \lambda_2 J_1(\lambda_2 \rho) - \frac{C}{a} \lambda_3 J_1(\lambda_3 \rho) \right] e^{i(qz+\omega t)} \tag{17a}$$

$$u_z = \left[ik \frac{A}{a} J_0(\lambda_1 \rho) + ik \frac{B}{a} J_0(\lambda_2 \rho) + \frac{C}{a} \lambda_3^2 J_0(\lambda_3 \rho) \right] e^{i(qz+\omega t)} \tag{17b}$$

In cylindrical polar coordinates the nonvanishing stress components of temperature rate-dependent thermoelastic material in the absence of the variable θ are given by

$$t_{rr} = \lambda \nabla \cdot \mathbf{u} + 2\mu \frac{\partial u_r}{\partial r} - \beta T - \nu \beta \dot{T} \tag{18a}$$

$$t_{rz} = \mu \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) \tag{18b}$$

If we calculate related quantities and put them in Eqs. (18*a*) and (18*b*) we get the stress components as

$$t_{rr} = \left\{ \lambda \left[-\frac{A}{a^2}(\lambda_1^2 + k^2)J_0(\lambda_1\rho) - \frac{B}{a^2}(\lambda_2^2 + k^2)J_0(\lambda_2\rho) \right] \right.$$

$$+ 2\mu \left[-\frac{A}{a^2}\lambda_1^2\left(J_0(\lambda_1\rho) - \frac{1}{\lambda_1\rho}J_1(\lambda_1\rho) \right) - \frac{B}{a^2}\lambda_2^2\left(J_0(\lambda_2\rho) \right. \right.$$

$$\left. \left. - \frac{1}{\lambda_2\rho}J_1(\lambda_2\rho) \right) - ik\frac{C}{a^2}\lambda_3^2\left(J_0(\lambda_3\rho) - \frac{1}{\lambda_3\rho}J_1(\lambda_3\rho) \right) \right]$$

$$\left. - \rho_0 c_1^2 \left[\frac{A}{a^2}(\Omega^2 - k^2 - \lambda_1^2)J_0(\lambda_1\rho) + \frac{B}{a^2}(\Omega^2 - k^2 - \lambda_2^2)J_0(\lambda_2\rho) \right] \right\} e^{i(qz+\omega t)}$$

$$t_{rz} = \mu \left[-2ik\lambda_1\frac{A}{a^2}J_1(\lambda_1\rho) - 2ik\lambda_2\frac{B}{a^2}J_1(\lambda_2\rho) - \frac{C}{a^2}\lambda_3(\lambda_3^2 - k^2)J_1(\lambda_3\rho) \right] e^{i(qz+\omega t)}$$

DISPERSION RELATION

If the lateral surface of the cylinder is held at constant temperature, the boundary conditions that must be satisfied by stress components and temperature are

$$t_{rr} = 0 \qquad t_{rz} = 0 \qquad T = 0 \qquad \text{for } \rho = 1 \text{ (or } r = a)$$

Utilizing these conditions, we get three homogeneous linear equations in A, B, C. In order to have a nontrivial solution to these equations, the determinant of the coefficient matrix must be equal to zero. This condition gives us the following relation:

$$(\Omega^2 - k^2 - \lambda_1^2)J_0(\lambda_1)\left\{ [\lambda\Omega^2 J_0(\lambda_2) + 2\mu(\Omega^2 - k^2)J_0(\lambda_2) \right.$$

$$\left. - 2\mu\lambda_2 J_1(\lambda_2)]\lambda_3(\lambda_3^2 - k^2)J_1(\lambda_3) + \left[J_0(\lambda_3) - \frac{1}{\lambda_3}J_1(\lambda_3) \right] 4\mu k^2\lambda_2\lambda_3^2 J_1(\lambda_2) \right\}$$

$$- (\Omega^2 - k^2 - \lambda_2^2)J_0(\lambda_2)\left\{ [\lambda\Omega^2 J_0(\lambda_1) + 2\mu(\Omega^2 - k^2)J_0(\lambda_1) - 2\mu\lambda_1 J_1(\lambda_1)] \right.$$

$$\left. \cdot \lambda_3(\lambda_3^2 - k^2)J_1(\lambda_3) + \left[J_0(\lambda_3) - \frac{1}{\lambda_3}J_1(\lambda_3) \right] 4\mu k^2\lambda_1\lambda_3^2 J_1(\lambda_1) \right\} = 0 \qquad (19)$$

Here Ω and k are dimensionless complex frequency and real wave number, respectively, λ_1^2 and λ_2^2 are the roots of Eq. (9), and $\lambda_3^2 = \gamma^2\Omega^2 - k^2$. This equation is the dispersion relation for the particular problem whose boundary conditions are given above. As it is not possible to obtain the roots of this equation analytically, we will give its numerical solutions. However, some analytical results can be obtained by assuming that the radius of cylinder is small. Then the Bessel functions can be expanded in power series in the radius a. Since the field equations are nondimensionalized, the quantity a is not seen explicitly in them. But if we remember the definitions of the dimensionless quantities we can see that smallness of the radius a corresponds to smallness of the dimensionless wave number k. Arranging the dispersion relation, we obtain the following form:

$$(\lambda_3^2 - k^2)^2(\lambda_2^2 - \lambda_1^2) + 4k^2\lambda_2\lambda_3(\Omega^2 - k^2 - \lambda_1^2)\frac{J_1(\lambda_2)}{J_0(\lambda_2)}\frac{J_0(\lambda_3)}{J_1(\lambda_3)}$$
$$- 4k^2\lambda_1\lambda_3(\Omega^2 - k^2 - \lambda_2^2)\frac{J_1(\lambda_1)}{J_0(\lambda_1)}\frac{J_0(\lambda_3)}{J_1(\lambda_3)} - 2\lambda_2(\lambda_3^2 + k^2)(\Omega^2 - k^2 - \lambda_1^2)\frac{J_1(\lambda_2)}{J_0(\lambda_2)}$$
$$+ 2\lambda_1(\lambda_3^2 + k^2)(\Omega^2 - k^2 - \lambda_2^2)\frac{J_1(\lambda_1)}{J_0(\lambda_1)} = 0$$

For small k, considering

$$\Omega = \frac{c}{c_1}k \qquad \lambda_1^2 = \left(\frac{c^2}{c_1^2} - 1\right)k^2 + O(k^3)$$
$$\lambda_2^2 = -k^2 - \frac{i\alpha c}{c_1}k + \frac{\varepsilon_2 c^2}{c_1^2}k^2 - \frac{i\varepsilon_1 c^2}{c_1^2}k^2 + \varepsilon_1\varepsilon_3\frac{c^2}{c_1^2}k^2 + O(k^3) \tag{20}$$

and expanding the Bessel functions, we obtain the following expression:

$$\gamma^2(\gamma^2 - 1)\frac{c^2}{c_1^2} - 3\gamma^2 + 4 + O(k) = 0 \tag{21}$$

where $\gamma^2 = c_1^2/c_2^2$ and c is the phase velocity. For $k = 0$ it is found that the phase velocity c is equal to c_0 regardless of the values of the parameters ε_1, ε_2, ε_3.

NUMERICAL SOLUTION

Because of the nature of the problem, we expect the roots of the dispersion equation to be complex. Using a complex root finding program, a few modes are calculated for the real dimensionless wave number k. Taking the real parts of the Ω's, we find the dimensionless phase velocity as

$$\frac{c}{c_0} = \frac{c_1}{c_0 k}\operatorname{Re}\Omega$$

Since pure thermal waves are detected in the some metals cooled down to 4.2

K, we made our calculations about this temperature. The material properties for copper at room temperature are as follows:

$$\rho_0 = 8.96 \times 10^{-3}\ \text{kg/cm}^3$$

$$\lambda = 7.55 \times 10^8\ \text{kg/cm} \cdot \text{s}^2$$

$$\mu = 3.86 \times 10^8\ \text{kg/cm} \cdot \text{s}^2$$

$$K = 3.98 \times 10^4\ \text{kg} \cdot \text{cm/K} \cdot \text{s}^3$$

$$c_v = 3.845 \times 10^6\ \text{cm}^2/\text{K} \cdot \text{s}^2$$

where ρ_0 is the density, λ and μ are Lamé constants, K is the thermal conductivity, and c_v is the specific heat. Considering that a 1 K decrease in temperature increases the elastic moduli of the material by about 0.03% [8], the Lamé constants and other quantities at 4.2 K are taken approximately as[†]

$$\lambda = 8.20 \times 10^8\ \text{kg/cm} \cdot \text{s}^2$$

$$\mu = 4.20 \times 10^8\ \text{kg/cm} \cdot \text{s}^2$$

$$K = 113 \times 10^4\ \text{kg} \cdot \text{cm/K} \cdot \text{s}^3$$

$$c_v = 9.1 \times 10^2\ \text{cm}^2/\text{K} \cdot \text{s}^2$$

To calculate the coupling factor ε_1 it is necessary to know the value of the thermal expansion coefficient $\bar{\alpha}$ at 4.2 K, because $\beta = (3\lambda + 2\mu)\bar{\alpha}$. At temperatures less than $\theta_D/50$, where θ_D is the Debye temperature for the material, the thermal expansion coefficient is taken as approximately $10^{-8}\ \text{K}^{-1}$ for copper [9]. The Debye temperature for copper is 339 K. From these values ε_1 is found to be 2.6×10^{-7} at low temperatures. In fact, ε_1 is a small parameter. There are no experimental values for ε_2 and ε_3. A thermodynamic inequality from the generalized thermoelasticity theory indicates how the values for ε_2 and ε_3 can be selected [5]. According to this thermodynamic inequality, the following expression must be positive:

$$\nu\rho_0 c_v - \rho_0\tau \geq 0$$

If we multiply this inequality with appropriate terms the dimensionless form can be written as

$$\varepsilon_2 \leq \alpha\varepsilon_3 \tag{22}$$

The dimensionless parameter α is definted in Eq. (10).

†While λ and μ are calculated approximately by extrapolation, K and c_v are obtained from measurements.

Let us first consider the case in which $\varepsilon_1 = 2.6 \times 10^{-7}$, $\varepsilon_2 = \varepsilon_3 = 1$, and $\alpha = 2$.[‡] Because the coupling factor ε_1 is a small quantity, it can be assumed that the coupling between elastic and thermal field equations can be neglected. This is shown in the dispersion equation. As a result we may expect that some modes related to the elastic behavior of the material are real, and others related to the thermal behavior of the material are complex. Indeed, some of the modes are real and others are complex. An interesting point about the complex roots is that their imaginary parts are constants. For this special case they are equal to 1. Real parts of Ω as a function of k are presented in Fig. 1, where thin solid lines show the real roots and thick solid lines show the complex roots. Although the modes seem to intersect each other, in reality they do not. Only projections on the plane $(k, \text{Re}\,\Omega)$ intersect. This situation can be seen in Fig. 2, the three-dimensional form of Fig. 1. Phase velocities for five modes are depicted in Fig. 3.

As a second case we take coupling factor $\varepsilon_1 = 2.6 \times 10^{-7}$, $\varepsilon_2 = \varepsilon_3 = 1$, and $\alpha = 4$. Because ε_1 is a small parameter, the same things can be said in this case as in the preceding case. When the modes are calculated, it is seen that some of them are real and others complex. However, in this case the imaginary parts of the complex roots are equal to 2 (this is explained below). Real parts of Ω versus k are presented in Fig. 4. As in the preceding case, thin solid lines are related to real roots and thick solid lines to complex roots. Although they seem to intersect, Fig. 5, which is the three-dimensional representation of Fig. 4, shows that this is not the case. Phase velocities as a function of k are presented in Fig. 6.

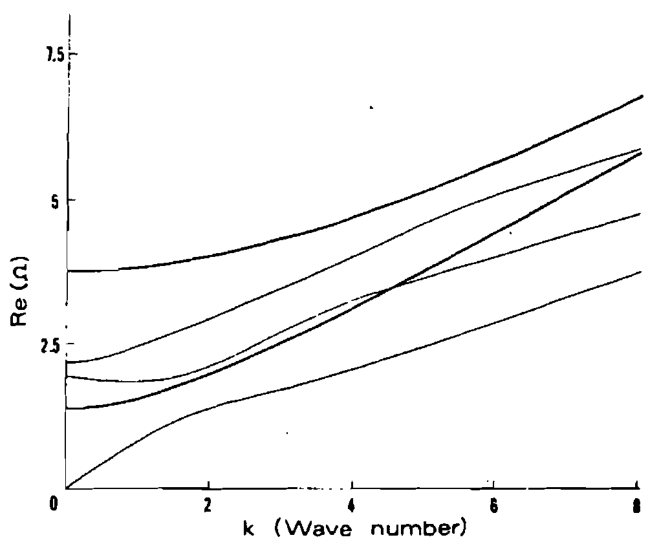

Fig. 1 Real part of Ω for five modes of the cylinder versus dimensionless wave number k, $\alpha = 2$, $\varepsilon_1 = 2.6 \times 10^{-7}$, $\varepsilon_2 = \varepsilon_3 = 1$.

‡Henceforth, in numerical calculations ε_2, ε_3, and α will be chosen arbitrarily, complying only with the constraint of Eq. (22), since to our knowledge no experimental values are available in the literature.

Assuming that the coupling factor ε_1 is a small quantity, if we expand the dispersion relation in power series in ε_1 we obtain the following equation:

$$-(\Omega^2 - \varepsilon_2\Omega^2 + i\alpha\Omega)J_0[(\varepsilon_2\Omega^2 - i\alpha\Omega - k^2)^{1/2}]\{(\lambda_3^2 - k^2)^2 J_0[(\Omega^2 - k^2)^{1/2}]J_1(\lambda_3)$$
$$- 2(\Omega^2 - k^2)^{1/2}(\lambda_3^2 + k^2)J_1[(\Omega^2 - k^2)^{1/2}]J_1(\lambda_3)$$
$$+ 4k^2(\Omega^2 - k^2)^{1/2}\lambda_3 J_1[(\Omega^2 - k^2)^{1/2}]J_0(\lambda_3)\} + O(\varepsilon_1) = 0 \tag{23}$$

Because of the smallness of ε_1, the dispersion equation becomes decoupled. The first two terms in Eq. (23) characterize thermal effects; the third term is the dispersion relation that corresponds to the elastic effect. This is why some roots are real and others complex. To understand why the imaginary parts of the complex roots are constant, let us look at the second expression in Eq. (23):

$$J_0[(\varepsilon_2\Omega^2 - i\alpha\Omega - k^2)^{1/2}] = 0$$

Because all the roots of $J_0(\ )$ become real for a given k and selected values of ε_2 and α, if we assume that Ω is a complex quantity, the imaginary part of the argument of J_0 must be equal to zero. This gives the following condition regardless of the values of the dimensionless wave number k:

$$\text{Im}\ \Omega = \frac{\alpha}{2\varepsilon_2}$$

which shows that the imaginary parts of the complex roots are constant. It is easy to see that if the material is chosen as a classical thermoelastic one, the parameter

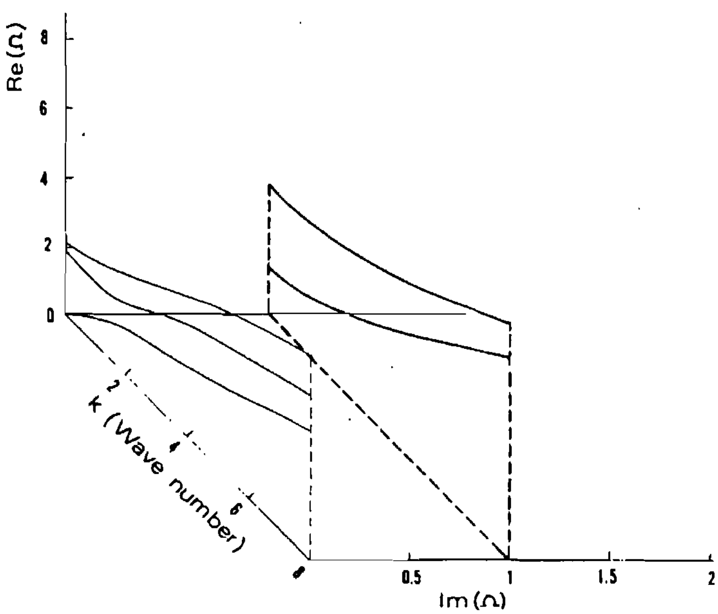

Fig. 2 Three-dimensional form of Fig. 1.

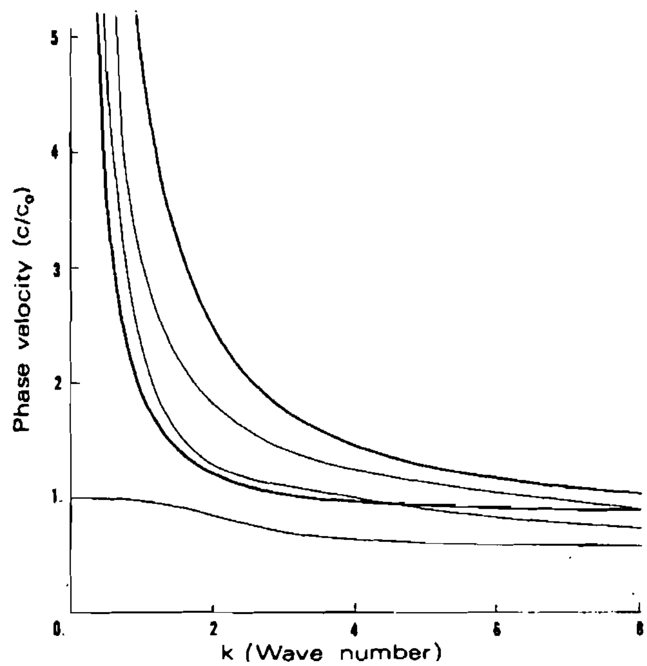

Fig. 3 Phase velocities versus k, $\alpha = 2$, $\varepsilon_1 = 2.6 \times 10^{-7}$, $\varepsilon_2 = \varepsilon_3 = 1$.

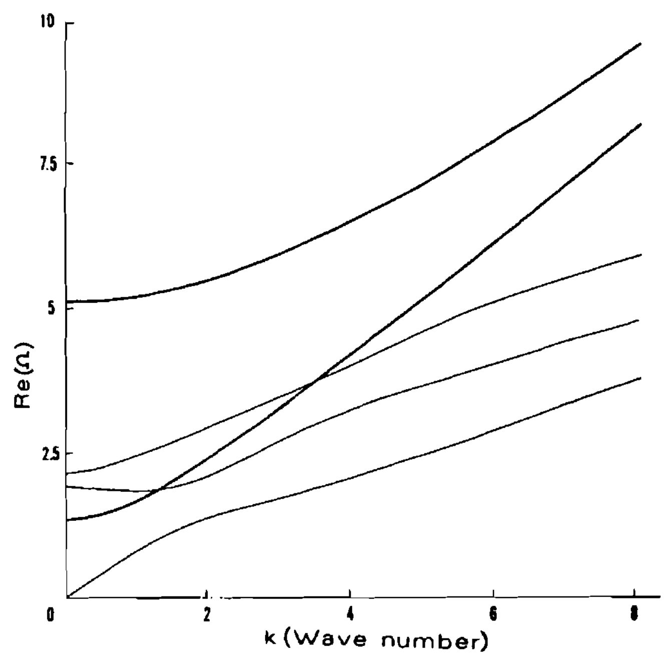

Fig. 4 Real part of Ω versus k, $\alpha = 4$, $\varepsilon_1 = 2.6 \times 10^{-7}$, $\varepsilon_2 = \varepsilon_3 = 1$.

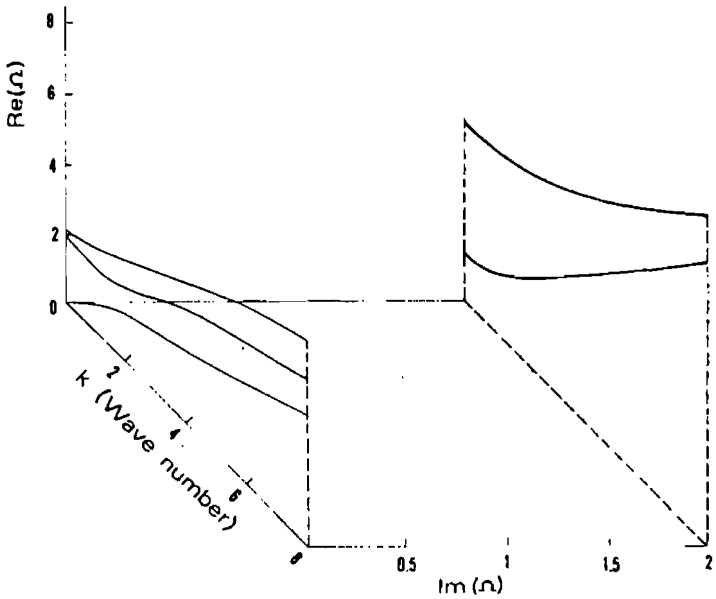

Fig. 5 Three-dimensional form of Fig. 4.

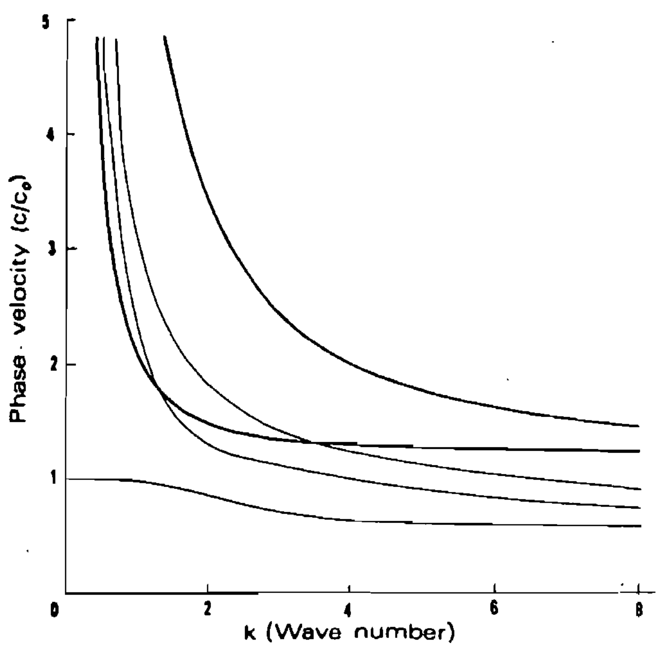

Fig. 6 Phase velocities versus k, $\alpha = 4$, $\varepsilon_1 = 2.6 \times 10^{-7}$, $\varepsilon_2 = \varepsilon_3 = 1$.

ε_2 is equal to zero. Then the roots Ω of $J_0[(-i\alpha\Omega - k^2)^{1/2}]$ become purely imaginary for all k.

As a third case, although it may seem not to agree with the physical facts, we take $\varepsilon_1 = 1$, $\varepsilon_2 = \varepsilon_3 = 1$, and $\alpha = 2$. Since ε_1 is taken as a large parameter, coupling between the elastic and thermal field equations is important and it can affect the characteristics of modes. Indeed, looking at the numerical results we can see that there are no real roots and the imaginary parts of the frequencies begin to rise; that is, dissipation becomes prominent. Real and imaginary parts of Ω versus k are shown in Figs. 7 and 8. However, elastic-like modes can be differentiated from thermal-like modes. In elastic-like modes dissipation is smaller, and in thermal-like modes attenuation is larger. This can be seen in Fig. 8, where thin solid lines show elastic-like modes and thick solid lines show thermal-like modes. Dissipation in the elastic-like modes is smaller than in the thermal ones. A three-dimensional representation of this case is presented in Fig. 9, where dashed lines show the projections of modes on the plane (k,Im Ω). Related phase velocity diagrams are presented in Fig. 10.

Finally, we take $\varepsilon_1 = 2$, $\varepsilon_2 = \varepsilon_3 = 2$, and $\alpha = 4$. As in the preceding case, dissipation effects begin to appear. In all modes dissipation is observable. However, elastic-like and thermal-like modes can be separated from each other because of the difference in their imaginary parts. Real and imaginary parts of Ω are depicted in Figs. 11 and 12, respectively. A three-dimensional representation of this case is shown in Fig. 13. Dashed lines in Fig. 13 show the projections of modes on the plane (k,Im Ω). Related phase velocity diagrams are shown in Fig. 14.

We can deduce from the above analysis that if the coupling parameter ε_1 is chosen as a small quantity the dispersion equation leads to two uncoupled modes. One of them is real and related to the elastic behavior, whereas the other is complex and represents the thermal behavior. If, contrary to the physical facts, we assume that the coupling factor ε_1 is a large parameter, dissipation appears in all modes. Because of the increased value of ε_1, real roots disappear and all modes become complex. But in some modes dissipation is smaller than in others, which means that although there is dissipation in all modes, elastic behavior dominates in some modes and thermal behavior in others.

CONCLUSIONS

Longitudinal wave propagation in an infinite, temperature rate-dependent thermoelastic cylinder has been studied, and the dispersion relation is obtained for a particular set of boundary conditions. Because of the difficulty of finding an analytical solution to the dispersion equation, a numerical solution is given within a certain interval of the real wave number. For various values of the parameters ε_1, ε_2, ε_3, a few numbers of modes have been obtained. The variations of frequencies and phase velocities with dimensionless wave number k have been presented.

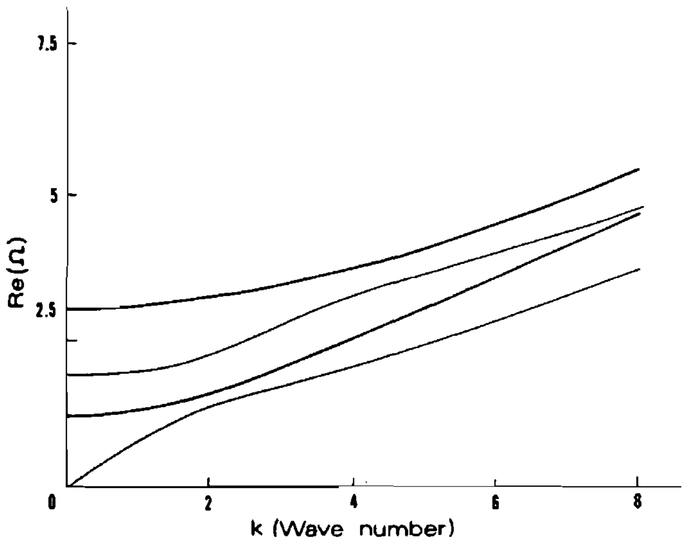

Fig. 7 Real parts of Ω versus k, $\alpha = 2$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$.

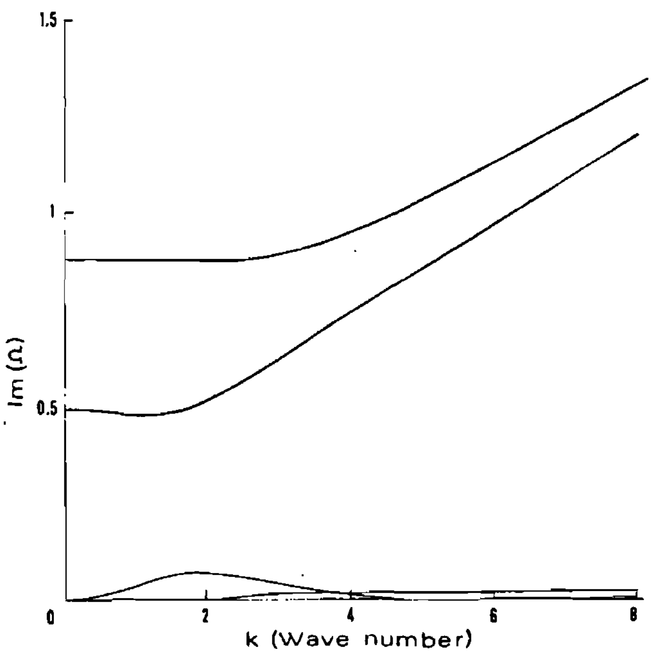

Fig. 8 Imaginary parts of Ω versus k, $\alpha = 2$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$.

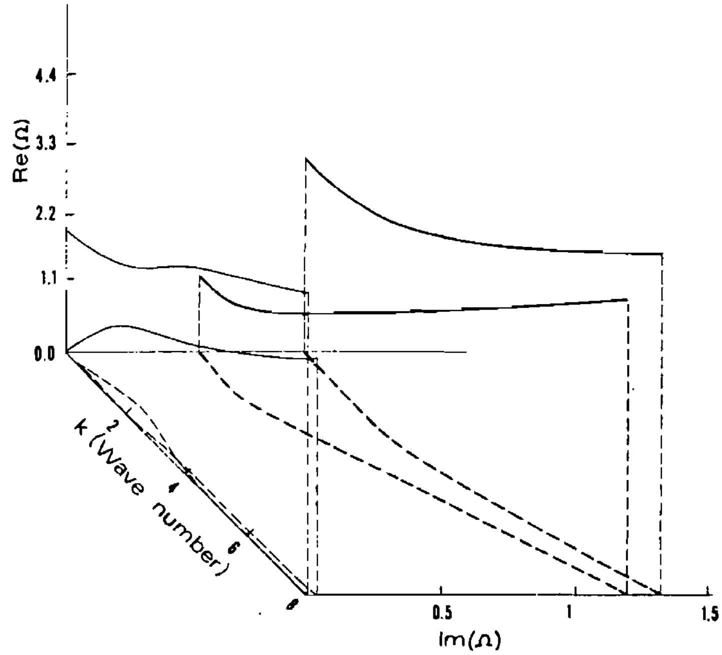

Fig. 9 Three-dimensional form of Figs. 7 and 8.

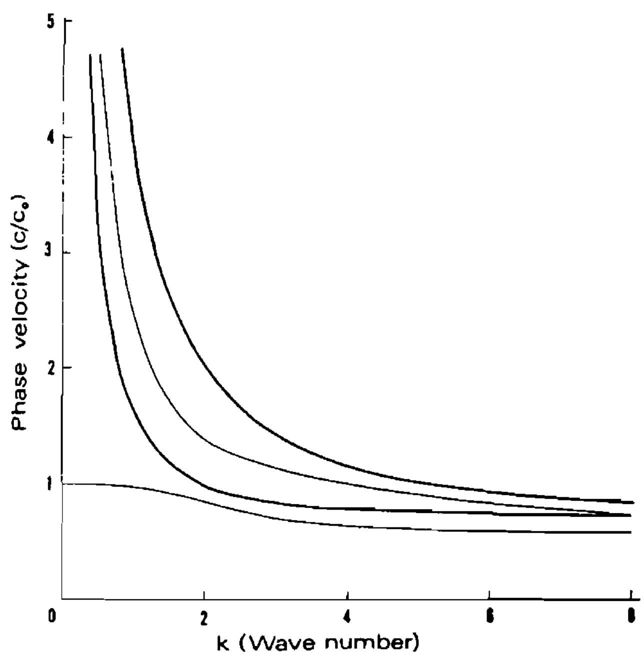

Fig. 10 Phase velocities, $\alpha = 2$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$.

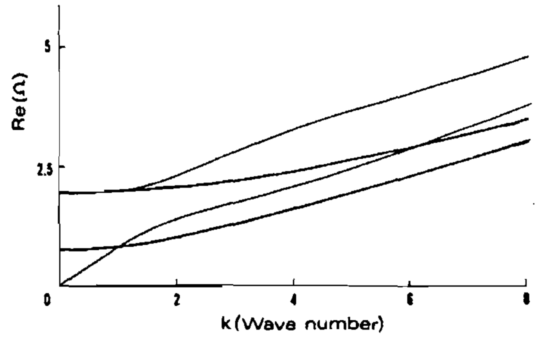

Fig. 11 Real parts of Ω versus k, $\alpha = 4$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 2$.

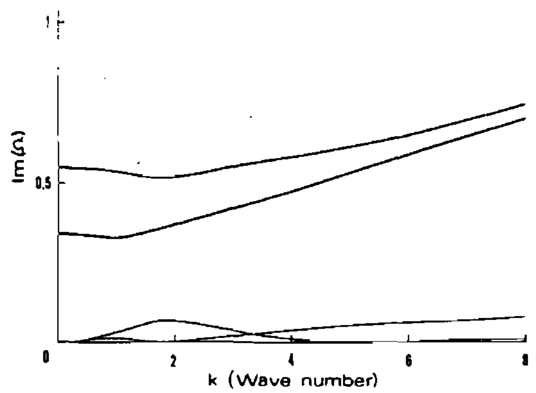

Fig. 12 Imaginary parts of Ω versus k, $\alpha = 4$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 2$.

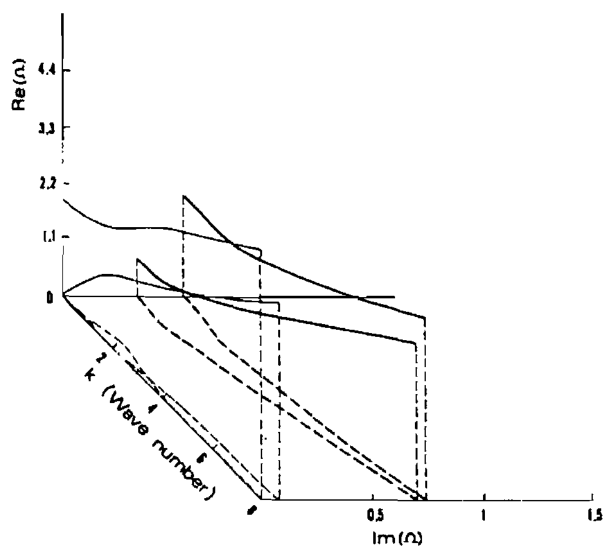

Fig. 13 Three-dimensional form of Figs. 11 and 12.

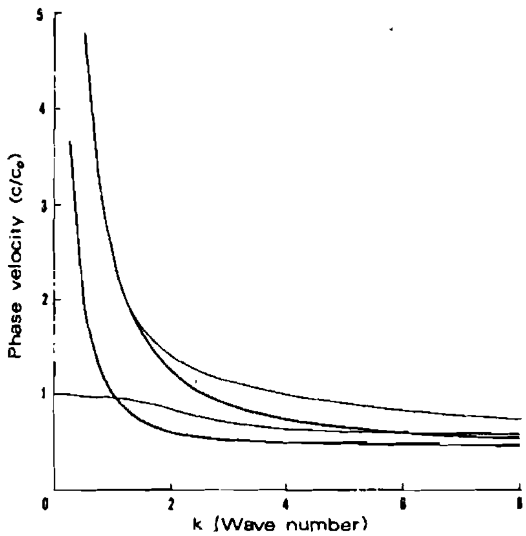

Fig. 14 Phase velocities versus k, $\alpha = 4$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 2$.

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